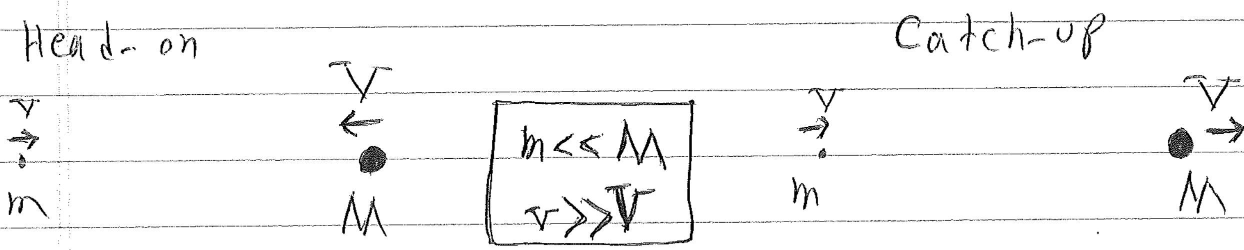
Lec 7: 02/08/2012

Particle Acceleration (Cont'd):

Fermi Acceleration:

Gas flowing into a compact object is often supersonic. This means that sound waves form discontinuous jumps across the resultant shock front. The diffusion of particles across the shock accelerates them and produces a power-law distribution, which is called the Fermi acceleration. The accelerating particles scatter off the random fluctuations in the magnetic field (which is a collisionless process). This mechanism constitutes the second electromagnetic acceleration scheme.

To see how random scattering of particles between molecular clouds in the interstellar medium can accelerate them, lets consider a simple one-dimensional version of the problem. We a large number of massive particles with mass M (the clouds) moving with the same speed V but randomly (i.e., back and forth). They collide with a small test particle of mass m, whose velocity is v. There are two types of collisions: head-on (h) and catch-up (c). Schematically, they look like as follows:



One can show by using the conservation of momentum and energy that the energy of the test particle changes by ΔE_h and ΔE_c in head-on and catch-up collisions respectively, where:

$$\Delta E_h = \frac{1}{2} m(v+2V)^2 - \frac{1}{2} m v^2 = 2mvV + 2mV^2$$

$$\Delta E_c = \frac{1}{2} m(v-2V)^2 - \frac{1}{2} m v^2 = -2mvV + 2mV^2$$

As seen, the test particle gains energy in a head-on collisions, and looses energy in a catch-up collision. To find the net increase in the energy, we should also include the probability for each type of collision. The probability is proportional to the collision rate, which is in turn proportional to the relative velocity of colliding particles. This results in:

$$P_h = \frac{v + V}{2v} \quad , \quad P_c = \frac{v - V}{2v}$$

Thus:

$$\langle \Delta E \rangle = P_h \Delta E_h + P_c \Delta E_c \Rightarrow \frac{\Delta E}{E} = 4 \left(\frac{V}{v} \right)^2$$

This is a second-order effect, hence to small to yield a significant energy increase. However, as we will see, a shock geometry improves the situation by producing first-order dependence of ΔE on the velecities.

Before turning to this, we can also find the energy spectrum resulting from this stochastic process by using the one-dimensional model. The energy distribution of particles satisfies the diffusion-loss equation:

$$\frac{dN(E,x,t)}{dt} = -\frac{\partial \phi_s}{\partial x} - \frac{\partial \phi_E}{\partial E} + \frac{\partial N(E,x,t)}{\partial t}$$

Here $N(E,x,t)\,dx\,dE$ is the number of particles between x and $x+dx$ with energy between E and $E+dE$ at time t. ϕ_s is the spatial flux and ϕ_E is the energy flux. The spatial flux can be written in terms of a diffusion coefficient D:

$$\phi_s = -D\frac{\partial N}{\partial x} \Rightarrow \frac{dN}{dt} = D\frac{\partial^2 N}{\partial x^2} - \frac{\partial \phi_E}{\partial E} + \frac{\partial N}{\partial t}$$

The energy flux is given by:

$$\phi_E = N\frac{dE}{dt}$$

Here $\frac{dE}{dt}$ is the rate of increase in the energy due to acceleration of particles. It is given by:

$$\frac{dE}{dt} = \langle \Delta E \rangle \nu = 4\nu\left(\frac{V}{v}\right)^2 E$$

ν is the frequency of collisions. The term $\frac{\partial N}{\partial t}$ represents explicit time variation in N. We can simplify the diffusion-loss equation by ignoring diffusion and setting $\frac{\partial N}{\partial t} = -\frac{N}{\tau}$, where τ is the characteristic time scale that particles spend in the acceleration zone. We then find the following equation:

$$\frac{dN}{dt} \approx -\frac{\partial}{\partial E}(N\alpha E) - \frac{N}{\tau} \qquad \alpha \equiv 4\nu\left(\frac{V}{v}\right)^2$$

In equilibrium $\frac{dN}{dt} = 0$, which gives rise to:

$$\frac{\partial(N\alpha E)}{\partial E} \approx -\frac{N}{\tau} \Rightarrow \alpha N + \alpha E\frac{\partial N}{\partial E} \approx -\frac{N}{\tau} \Rightarrow \frac{\partial N}{\partial E} \approx -\left(1+\frac{1}{\alpha\tau}\right)\frac{N}{E}$$

We find a final solution to this equation:

$$N(E) \approx N_0 E^{-\left(1+\frac{1}{\alpha\tau}\right)}$$

This is a remarkable result demonstrating that the energy spectrum is a power law function, as seen in the case of cosmic rays and many high-energy sources. The spectral index $-\left(1+\frac{1}{\alpha\tau}\right)$ may vary slightly from location to location.